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An improved auto-tuning scheme for PID controllers

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1. Introduction

In spite of various advancements in process control techniques, up until now, PID controllers have been very popular in industrial close-loop control [1–4]. An extensive survey on the regulatory controllers used in refinery, chemical, pulp, and paper industries reveals that 97% of them are of PID structure; even sophisticated control techniques also embed PID algorithms at the lowest level [3]. Simplicity, applicability, and ease of implementation have led to its wide acceptance [4]. But for many of them, performance is quite poor due to, among other factors, inadequate tuning of the controller parameters [5-7]. Though many tuning methods have been proposed for PID controllers over the past half century [8], so far no scheme has replaced the simple ZN tuning rules [9] in terms of familiarity and ease of use to start with [4]. The close-loop ZN tuning [9] is one of the most popular methods to obtain reasonably good initial settings for PID controllers [4,10]. However, ZNPIDs are found to perform quite satisfactorily for first-order processes, but they usually fail to provide acceptable performance for high-order and nonlinear processes [7,10,12] due to large overshoots and poor load regulation.

To overcome such drawbacks several tuning schemes are proposed [7,10–17]. In [13] a time response based design methodology is presented for PID controllers. Depending on the magnitude of normalized dead-time, three types of tuning rules are

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ABSTRACT

An improved auto-tuning scheme is proposed for Ziegler-Nichols (ZN) tuned PID controllers (ZNPIDs), which usually provide excessively large overshoots, not tolerable in most of the situations, for high-order and nonlinear processes. To overcome this limitation ZNPIDs are upgraded by some easily interpretable heuristic rules through an online gain modifying factor defined on the instantaneous process states. This study is an extension of our earlier work [Mudi RK., Dey C. Lee TT. An improved auto-tuning scheme for PI controllers. ISA Trans 2008; 47: 45–52] to ZNPIDs, thereby making the scheme suitable for a wide range of processes and more generalized too. The proposed augmented ZNPID (AZNPID) is tested on various highorder linear and nonlinear dead-time processes with improved performance over ZNPID, refined ZNPID (RZNPID), and other schemes reported in the literature. Stability issues are addressed for linear processes. Robust performance of AZNPID is observed while changing its tunable parameters as well as the process dead-time. The proposed scheme is also implemented on a real time servo-based position control system. © 2009 ISA. Published by Elsevier Ltd. All rights reserved.

> proposed for processes with time-delay ranging from zero to large values. Robust and optimal setting for PID controllers is proposed in [14], where optimization has resulted in a couple of tuning rules for stable, oscillating, and non-oscillating plants. A simple method for the tuning of PID controllers for integrating processes with dead-time is suggested in [15]. It is based on matching the coefficient of the corresponding powers of *s* in the numerator and that in the denominator of the close-loop transfer function. A similar method for tuning PID controllers [16] for first-order plus dead-time (FOPDT) processes is presented with a performance comparable to that of a ZNPID. In [17] a PID controller is designed based on transient performance specification with monotonic step response. All these suggested tuning schemes for PID controllers are essentially applicable for linear systems.

> For managing difficult tasks in nonlinear process control, auto-tuning is a desirable feature and almost every industrial PID controller provides it nowadays [4]. Various auto-tuning schemes [18-24] are reported in the literature. A gain scheduling scheme [19] is proposed to continuously update the proportional and integral gains depending on the error signal. A combined least-squares estimation and search technique [20] is used for the automatic tuning of ZNPIDs. To ensure the robustness of performance and higher stability for FOPDT processes, a PID controller with dual adaptive loops is presented in [21]. The first adaptive loop makes online tuning of the PID controller to ensure stability before updating the nominal model, and the second loop identifies the changes to the nominal model and retunes the controller accordingly.

> To achieve improved robustness and better transient response, back-stepping based adaptive PID control is proposed [22], which

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Fig. 1. Block diagram of the proposed AZNPID.

leads to a PD adaptive controller for linear minimal phase processes. A robust self-tuning PID controller [23] is developed for nonlinear systems with a high gain preload relay kept in series. The chattering signal (considered as a naturally occurring signal) is used for tuning and retuning the PID controller under different operating conditions. Amplitude dependent gain adjustment scheme is used in [24] to obtain the ultimate point of frequency response with better accuracy compared to relay feedback technique. Based on the normalized dead-time and normalized gain of the process a guideline is provided for the selection of suitable control algorithms, and refinements of ZN tuning rules for PI and PID controllers in order to achieve enhanced performances [7].

Dynamics of industrial processes are not completely known and are subjected to changes under various operating conditions. To obtain the desired response an online gain modification scheme is proposed for ZNPIs [12], which simultaneously adjusts both proportional and integral gains depending on the instantaneous error (*e*) and change of error (Δe) of the controlled variable. It has already been mentioned that the present study is an endeavor to extend the basic auto-tuning strategy for ZNPIs [12] to ZNPIDs, thereby making the scheme more general and applicable for a wide range of processes. Similar knowledge based online tuning schemes with fuzzy *If-Then* rules are used to adjust the output scaling factor (equivalent to the overall gain of the controller) of self-tuning fuzzy logic controllers [25,26], and also to update the parameters of conventional non-fuzzy PID controllers [27] based on the instantaneous process states (i.e., *e* and Δe).

While running a plant in manual mode, an operator generally adjusts the controller gains according to the current process trend to attain the desired response. The basic idea behind such gain manipulation strategy is that, when the process variable is moving away from the set-point, controller takes aggressive action to bring it back to the desired value as soon as possible. On the other hand, when the process is moving fast towards the set-point, control action is reduced to restrict the potential overshoot and undershoot in subsequent operating phases. In the proposed AZNPID, we try to realize the above gain modification strategy with the help of some simple heuristic rules incorporating an online gain updating factor α , defined on the normalized *e* and Δe . Here, proportional, integral, and derivative gains of AZNPID are adjusted towards improving the process response during set-point change as well as load disturbance.

The performance of AZNPID is tested for several secondand third-order linear and nonlinear dead-time processes and compared with those of ZNPID [9], RZNPID [10], AZNPI [12], and that proposed by Luyben [11] (LPID) in terms of a number of performance indices – percentage overshoot ((OS), risetime (t_r), settling-time (t_s), integral-absolute-error (*IAE*), and integral-time-absolute-error (*ITAE*). Performance analysis reveals that the proposed AZNPID is capable of providing an improved overall performance both in transient and steady state conditions. Robustness of AZNPID is tested by varying controller parameters as well as process dead-time. Stability issues of this nonlinear controller (AZNPID) are addressed for linear processes. The proposed AZNPID is successfully implemented on a real time servobased position control system. The rest of the paper is divided into three parts. Section 2 presents the proposed controller design along with its tuning strategy and stability issues. Simulation results for various linear and nonlinear dead-time processes as well as the real time implementation of AZNPID are illustrated in Section 3. We conclude in Section 4.

2. The proposed controller

2.1. Design of AZNPID

The simplified block diagram of the proposed PID controller is shown in Fig. 1. It shows that the gain updating factor α , a function of the process error (e) and change of error (Δe) continuously adjusts the parameters of a ZNPID. Fig. 1 indicates that the starting point of the AZNPID for a given process is its corresponding ZNPID, which means initial settings of the proposed PID auto-tuner are based on ZN tuning rules. Each of such ZN tuned parameters of AZNPID (i.e., proportional, integral, and derivative gains) is updated online by the single modifying factor α through some simple relations.

Let the discrete form of a conventional ZNPID be described as

$$u^{c}(k) = K_{p} \left[e(k) + \frac{\Delta t}{T_{i}} \sum_{i=0}^{k} e(i) + \frac{T_{d}}{\Delta t} \Delta e(k) \right]$$

= $K_{p}e(k) + K_{i} \sum_{i=0}^{k} e(i) + K_{d} \Delta e(k).$ (1)

In Eq. (1), $u^c(k)$ is the control action at *k*th sampling instant, K_p is the proportional gain, $K_i = K_p(\Delta t/T_i)$ is the integral gain, and $K_d = K_p(T_d/\Delta t)$ is the derivative gain where T_i is the integral time, T_d is the derivative time, and Δt is the sampling interval. K_p , T_i , and T_d are calculated according to ZN ultimate cycle tuning rules (i.e., $K_p = 0.6 k_u$, $T_i = 0.5 t_u$, and $T_d = 0.125 t_u$, where k_u and t_u are the ultimate gain and ultimate period respectively). Here, e(k) and $\Delta e(k)$ are expressed as

$$e(k) = r - y(k), \qquad (2)$$

$$\Delta e(k) = e(k) - e(k-1),$$
(3)

when *r* is the set-point, and y(k) is the process output. The proposed gain updating factor α is defined by

$$\alpha(k) = e_N(k) \times \Delta e_N(k). \tag{4}$$

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Fig. 2. Variation of α with e_N and Δe_N .

Here
$$e_N(k) = \frac{e(k)}{|r|},$$
 (5)

and
$$\Delta e_N(k) = e_N(k) - e_N(k-1)$$
 (6)

are the normalized values of e(k) and $\Delta e(k)$ respectively. From Eq. (4), without loss of generality, it may be assumed that the possible variation of α will lie in the range [-1, 1] for all close-loop stable processes.

In our proposed AZNPID, K_p , K_i , and K_d will be continuously modified by the gain updating factor α with the following simple heuristic relations:

 $K_{p}^{m}(k) = K_{p} \left(1 + k_{1} \left| \alpha(k) \right| \right), \tag{7}$

$$K_i^m(k) = K_i \left(0.3 + k_2 \alpha(k) \right), \tag{8}$$

$$K_d^m(k) = K_d \left(1 + k_3 |\alpha(k)| \right).$$
(9)

Thus, from Eqs. (1) and (7)–(9), AZNPID can be expressed as

$$u^{a}(k) = K_{p}^{m}(k)e(k) + K_{i}^{m}(k)\sum_{i=0}^{k}e(i) + K_{d}^{m}(k)\Delta e(k), \qquad (10)$$

where $K_p^m(k)$, $K_i^m(k)$, and $K_d^m(k)$ are the modified proportional, integral, and derivative gains respectively at *k*th instant, and $u^a(k)$ is the corresponding control action. In Eqs. (7)–(9), k_1 , k_2 , and k_3 are three positive constants, which will make the required variations in K_n^m , K_i^m , and K_d^m around their respective initial values.

The objective of the proposed auto-tuning scheme is that, subsequent to any set-point change or load disturbance, the three parameters of AZNPID (i.e., K_p^m , K_i^m , and K_d^m) will be continuously adjusted by the nonlinear updating factor α in order to have a quick recovery of the process during both the set-point change and load variation without a large number of oscillations. Such real time nonlinear gain variations are introduced towards

achieving an enhanced control performance. Eqs. (7)–(9) indicate that compared to ZNPID, in AZNPID both the proportional and derivative gains, i.e., K_p^m and K_d^m are increased throughout the entire operating cycle, though may not be in the same proportion (due to difference in the values of k_1 and k_3), while the integral gain (K_i^m) is either increased or decreased from its initial setting depending on the operating phases.

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Fig. 2 shows the highly nonlinear variation of α . Therefore, unlike the linear control surface of ZNPID (Fig. 3(a)), AZNPID has a highly nonlinear control surface as depicted in Fig. 3(b) due to its nonlinear gain variations according to Eqs. (7)–(9). Note that in spite of its nonlinear gain variation, the basic PID structure is preserved in AZNPID. Therefore, an existing PID controller can be easily modified into the proposed form just by incorporating α . Moreover, the present gain modification scheme is independent of any process parameter or any performance index; it depends only on the recent process states.

2.2. Tuning strategy

While designing AZNPID, the following major points are taken into consideration to provide the appropriate control action in different operating phases. For a better understanding, typical close-loop response of an under-damped second-order process and its corresponding variation of α are illustrated in Fig. 4.

- (i) When the process is far from the set-point and moving fast towards it (e.g., points A, C, or F in Fig. 4), proportional gain should be reasonably large to reach the set-point quickly but the integral gain should be small enough to prevent the large accumulation of control action, which may result in a large overshoot or undershoot in future. At the same time, to reduce oscillations derivative gain should be increased for higher damping. Observe that, in such transient phases *e* and Δe are of opposite signs. Therefore, α becomes negative according to Eq. (4), which will make both the proportional and derivative gains higher, and integral gain lower than their corresponding initial values (i.e., $K_p^m > K_p, K_d^m > K_d$, and $K_i^m < 0.3K_i$) as indicated by Eqs. (7)–(9). Thus, our proposed gain adaptive rules (Eqs. (7)–(9)) try to adjust the parameters of AZNPID towards reducing the overshoot and/or undershoot, and oscillation in the process response.
- (ii) When the process is moving further away from the setpoint (e.g., points B, D, or E in Fig. 4), increased proportional, derivative as well as integral gains are expected to bring back the process variable to its desired value quickly. Under such situations, both *e* and Δe will have the same sign, thereby making α positive (Eq. (4)), which in turn makes all the gain parameters of AZNPID (i.e., K_p^m, K_d^m , and K_i^m) larger than their



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Fig. 4. Typical close-loop response of an under-damped second-order process (above) and its corresponding variation of the gain updating factor α (below).

respective initial values according to Eqs. (7)–(9). As a result, the control action becomes more aggressive (i.e., $u^a > u^c$ according to Eqs. (1) and (10)), which will try to restrict further deterioration of such situations. Therefore, AZNPID satisfies the need for a relatively strong control action to improve the process recovery.

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From the above discussion it is evident that our proposed autotuning scheme always attempts to modify AZNPID parameters (proportional, integral, and derivative gains) in the right directions to generate required control action in different transient phases for providing improved performances under both set-point change and load disturbance. Of course, depending on the type of response desired to achieve, suitable values of k_1 , k_2 , and k_3 are to be selected by the designer either from the knowledge about the process to be controlled or through trial and error.

Alternatively, for a given process suitable optimization (multiobjective optimization seems to be more appropriate) tools may be applied to find their best possible values with a view to achieving the desired performance with some pre-specified performance indices in the transient response. Although, satisfying stringent requirements for transient response while maintaining the robustness of the system seems to be the most difficult task in designing a controller [14,28]. In general, many optimization schemes do not work well. Optimization results are highly dependent on initializations. Above all, the problem of trapping in local minima is always there. Even after optimization the design goal may be underachieved. Therefore, the best optimization algorithm for a particular system needs to be chosen carefully. But a typical designer or a practicing engineer may not be familiar with various optimization techniques and associated software tools.

In this study, we do not use any optimization scheme. The values of k_1 , k_2 , and k_3 (i.e., $k_1 = k_2 = 1$, $k_3 = 12$) chosen for AZNPID are found empirically through extensive simulation experiments on a wide range of linear and nonlinear processes. Moreover, the same values are used for all the processes in our

simulation as well as real time experiments, thereby suggesting the proposed auto-tuning scheme as a generalized one rather than a specific optimization based design.

In this regard, it is worth mentioning that our proposed AZNPID apparently adds three new tuning parameters k_1 , k_2 , and k_3 over ZNPID. However, while setting their appropriate values, first we consider $k_1 = k_2 = 1$, and then, we chose the value of $k_3 = 12$ based on extensive simulation results as mentioned above. So, we may not take into account the tuning of two additional parameters k_1 and k_2 . But they are incorporated in Eqs. (7) and (8) only to make the gain modifying rules for K_p and K_i (i.e., Eqs. (7) and (8)) more flexible. Therefore, the proposed controller actually needs tuning of a single additional parameter k_3 .

2.3. Stability

The proposed AZNPID is a nonlinear controller due to the nonlinear variation of α . We know that the stability analysis for nonlinear systems is not straightforward. However, here we make an attempt to study the relative stability for the linear processes under close-loop control with AZNPID by calculating the gain margin (GM) and phase margin (PM) for two boundary values of α , i.e., α_{max} and α_{min} . Here, we find these two boundary values when the process shows sustained oscillation under proportional control, i.e., the process is at the verge of instability. Using these extreme values (α_{max} and α_{min}), relative stability margins, i.e., GM and PM along with their corner frequencies are evaluated for the respective linear process under ZNPID and AZNPID. In the result section, it will be shown that all the linear close-loop systems under AZNPID have good stability margins.

Moreover, to verify the stability robustness for the proposed controller, Kharitonov's method [29] has been used with the boundary values of α , i.e., α_{max} and α_{min} . Regardless of the degree of polynomials only four polynomials (Kharitonov's polynomials) has to be tested to check whether the close-loop system is robustly stable or not. The system will be robustly stable if all the roots of the



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(b) Responses and control actions for the second-order linear process with L = 0.3 s.

polynomials have negative real parts. In the following section, we shall justify the stability robustness of AZNPID for linear processes.

3. Results

Effectiveness of the proposed scheme is verified through simulation experiments on second- and third-order processes [5]. We have also tested our proposed AZNPID on a real time digital servo system of Feedback Instruments Limited [30]. In addition to response characteristics, performance of the proposed AZNPID is compared with those of ZNPID, RZNPID, LPID, and AZNPI with respect to a number of performance indices, such as %OS, *t*_r, *t*_s, *IAE*, and *ITAE*. For each of the linear process under AZNPID its stability margins (GM and PM) and Kharitonov's polynomials are evaluated for stability analysis.

The values of tuning parameters k_1 , k_2 , and k_3 are chosen as $k_1 = k_2 = 1$, and $k_3 = 12$ (these are empirical values as previously mentioned) for all the examples. To investigate the sensitivity of these tuning parameters on the performance of AZNPID, $\pm 25\%$ perturbations are applied on their above mentioned values. In order to study the robust performance of AZNPID the process dead-time has been increased by a considerable amount from its nominal value. Now we present the detailed performance analysis for linear and nonlinear processes including the real time implementation of a practical servo system.

3.1. Linear processes

For our simulation experiments, we consider the following linear processes used in [4,5]:

3.1.1. Second-order linear process

Transfer function of the process is given by

$$G_p(s) = \frac{e^{-Ls}}{(1+s)^2}.$$
 (11)

Fig. 5(a) shows the response characteristics of the process in (11) with L = 0.2 s under ZNPID, RZNPID, and AZNPID. Various performance indices for these PID controllers along with LPID [11], and AZNPI (proposed earlier [12]) are provided in Table 1a. Fig. 5(a) exhibits considerably improved performance of AZNPID over ZNPID, even better than RZNPID, specifically during load disturbance. Table 1a clearly indicates the superiority of AZNPID over ZNPID, RZNPID, LPID, and AZNPI. For examples, %OS has been drastically reduced from more than 60% to below 1%, and t_s is reduced over 60% with respect to ZNPID. Nature of variation in control actions for different PID controllers corresponding to responses are shown in Fig. 5(a). Table 1a clearly reveals the robust performance of the proposed scheme against considerable perturbations ($\pm 25\%$) on the tuning parameters k_1 , k_2 , and k_3 .

To further study the robustness of the proposed controller, a 50% higher value of dead-time, i.e., L = 0.3 s is considered with the same controller setting as that of L = 0.2 s. Corresponding responses and the related control actions are shown in Fig. 5(b), which also shows the same level of improvement in AZNPID compared to ZNPID and RZNPID. Note that, even with this increased dead-time overall performance of AZNPID is much better than that of AZNPI (Table 1a), though the %OS in AZNPI (about 6%) is slightly less than that of AZNPID. Table 1b presents the gain margin (GM) and phase margin (PM) of AZNPID for two possible values of α

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Fig. 5.

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Table 1a

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Performance analysis of the second-order linear process: $G_p(s) = \frac{e^{-Ls}}{(1+s)^2}$.

L		ZNPID	RZNPID	LPID	AZNPI	AZNPID							
						$k_1 = 1$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 1$	$k_1 = 1$
						$k_2 = 1$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1$	$k_2 = 1.25$	$k_2 = 1$
						$k_3 = 12$	$k_3 = 15$	$k_3 = 9$	$k_3 = 9$	$k_3 = 15$	$k_3 = 12$	$k_3 = 12$	$k_3 = 15$
0.2 s	% <i>OS</i>	60.61	11.40	0.00	4.04	0.56	2.26	2.22	2.05	2.18	0.42	0.78	2.22
	$t_r(s)$	1.00	1.50	1.60	2.30	1.30	2.00	1.20	1.20	1.98	1.30	1.40	1.90
	$t_s(s)$	4.00	3.90	10.90	2.60	1.60	2.30	1.50	1.50	2.20	1.60	1.70	2.20
	IAE	2.56	2.23	2.18	2.16	1.69	1.74	1.70	1.69	1.75	1.68	1.70	1.74
	ITAE	11.01	10.06	8.96	9.32	7.28	7.33	7.47	7.38	7.40	7.25	7.28	7.36
	%OS	85.43	26.40	6.60	5.98	12.27	7.58	17.10	16.98	7.59	13.08	11.46	7.59
036	$t_r(s)$	0.90	1.30	1.30	2.40	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10
0.5 5	$t_s(s)$	8.10	5.60	6.60	5.90	4.20	4.20	4.40	4.40	4.20	4.30	4.20	4.20
	IAE	4.11	3.34	2.58	3.04	2.23	2.20	2.30	2.30	2.19	2.24	2.22	2.19
	ITAE	29.64	26.76	17.07	26.58	15.70	15.61	16.08	16.08	15.54	15.78	15.65	15.58

Table 1b

Stability and robustness analysis for the second-order linear process: $G_p(s) = \frac{e^{-0.2s}}{(1+s)^2}$.

	ZNPID	AZNPID		Kharitonov's Polynomials for AZNPID with α_{max} and α_{min}
		$\alpha_{\rm max}$	α_{\min}	
GM (dB) PM (deg)	Inf (Inf rad/s) 45.1 (1.71 rad/s)	Inf (Inf rad/s) 72.5 (2.76 rad/s)	Inf (Inf rad/s) 72.9 (2.57 rad/s)	$s^{4} + 7s^{3} + 27.6s^{2} + 22.9s + 3.25 = 0;$ $s^{4} + 7s^{3} + 25.5s^{2} + 23.45s + 7.2 = 0;$ $s^{4} + 7s^{3} + 25.5s^{2} + 22.9s + 7.2 = 0;$ $s^{4} + 7s^{3} + 27.6s^{2} + 23.45s + 3.25 = 0$

Table 2a

Performance analysis of the second-order marginally stable process: $G_p(s) = \frac{e^{-Ls}}{s(s+1)}$.

L		ZNPID	RZNPID	LPID	AZNPI	AZNPID										
						$k_1 = 1$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 1$	$k_1 = 1$			
						$k_2 = 1$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1$	$k_2 = 1.25$	$k_2 = 1$			
						$k_3 = 12$	$k_3 = 15$	$k_3 = 9$	$k_3 = 9$	$k_3 = 15$	$k_3 = 12$	$k_3 = 12$	$k_3 = 15$			
	%OS	79.40	46.09	23.47	63.88	27.08	25.96	28.76	28.71	26.06	27.16	26.96	26.01			
0.2 s	$t_r(s)$	1.30	1.60	1.80	1.80	1.60	1.60	1.50	1.50	1.60	1.60	1.60	1.60			
	$t_s(s)$	8.70	8.40	11.10	17.90	9.60	9.80	9.40	9.30	9.90	9.60	9.60	9.90			
	IAE	4.43	3.78	3.85	7.01	3.38	3.39	3.38	3.37	3.39	3.37	3.38	3.39			
	ITAE	38.01	35.47	38.36	87.59	30.18	30.16	30.20	30.11	30.24	30.10	30.21	30.20			
	%OS	98.60	60.24	29.43	72.93	31.58	28.16	35.54	35.55	28.26	31.94	31.19	28.21			
030	$t_r(s)$	1.20	1.50	1.70	1.70	1.40	1.50	1.40	140	1.50	1.40	1.40	1.50			
0.3 S	$t_s(s)$	12.70	10.80	11.00	Unstable	9.40	9.80	9.00	9.00	9.80	9.30	9.40	9.80			
	IAE	5.98	5.04	4.07	9.28	3.59	3.58	3.60	3.59	3.59	3.59	3.59	3.58			
	ITAE	61.25	56.72	43.30	134.83	34.44	34.38	34.53	34.43	34.46	34.37	34.47	34.42			

Table 2b

Stability and robustness analysis for the second-order marginally stable process: $G_p(s) = \frac{e^{-0.2s}}{s(s+1)}$.

	ZNPID	AZNPID		Kharitonov's Polynomials for AZNPID with α_{max} and α_{min}
		$\alpha_{\rm max}$	$lpha_{ m min}$	
GM (dB) PM (deg)	Inf (Inf rad/s) 32.3 (1.26 rad/s)	Inf (Inf rad/s) 65.2 (1.86 rad/s)	Inf (Inf rad/s) 64.4 (1.79 rad/s)	$s^4 + 6s^3 + 15.1s^2 + 11.2s + 1.25 = 0;$ $s^4 + 6s^3 + 14.3s^2 + 11.3s + 2.25 = 0;$ $s^4 + 6s^3 + 14.3s^2 + 11.2s + 2.25 = 0;$ $s^4 + 6s^3 + 15.1s^2 + 11.3s + 1.25 = 0$

 $(\alpha_{\max} \text{ and } \alpha_{\min})$ obtained under sustained oscillation of the process, i.e., while the system is at the verge of instability. Table 1b also clearly reveals that the relative stability has been improved under

AZNPID in comparison with ZNPID. Stability robustness is verified from the negative roots of Kharitonov's polynomials (Table 1b) formed with two boundary values α_{max} and α_{min} .

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(a) Responses and control actions for the second-order marginally stable process with L = 0.2 s.



Time(sec)

Fig. 6.

Table 3a Performance analysis of the third-order linear process: $G_p(s) = \frac{1-\beta s}{(1+s)^3}$.

β		ZNPID	RZNPID	LPID	AZNPI	AZNPID							
						$k_1 = 1$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 1$	$k_1 = 1$
						$k_2 = 1$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1$	$k_2 = 1.25$	$k_2 = 1$
						$k_3 = 12$	$k_3 = 15$	$k_3 = 9$	$k_3 = 9$	$k_3 = 15$	$k_3 = 12$	$k_3 = 12$	$k_3 = 15$
	%OS	44.15	6.73	0.00	42.40	2.05	0.29	3.80	4.39	0.00	2.05	2.63	0.29
0.1	$t_r(s)$	2.00	3.11	29.98	2.10	2.51	2.60	2.42	2.33	2.69	2.51	2.51	2.65
	$t_s(s)$	10.85	9.47	29.98	-	16.38	16.24	16.34	17.07	15.60	16.61	16.38	16.01
	IAE	3.44	3.18	4.47	6.72	3.07	3.05	3.10	3.12	3.05	3.08	3.08	3.05
	ITAE	42.46	40.51	55.06	103.40	41.10	40.93	41.29	41.87	40.40	41.27	41.21	40.66
	%OS	54.68	12.57	0.00	53.51	7.89	5.56	9.65	10.82	4.39	7.89	7.89	4.97
0.2	$t_r(s)$	1.99	2.92	29.99	2.19	2.28	2.28	2.23	2.19	2.46	2.23	2.28	2.37
0.2	$t_s(s)$	14.12	13.80	29.99	-	15.41	15.23	15.41	17.16	14.82	15.64	15.37	15.09
	IAE	4.27	3.74	4.55	8.57	3.32	3.28	3.37	3.40	3.25	3.33	3.33	3.26
	ITAE	56.83	53.40	57.48	134.30	46.27	45.95	46.59	47.28	45.29	46.47	46.41	45.62

3.1.2. Second-order marginally stable process

$$G_p(s) = \frac{e^{-Ls}}{s(s+1)}.$$
 (12)

Fig. 6(a) and (b) depict the responses and the related control actions of this integrating process with L = 0.2 s and 0.3 s, respectively and detailed performance comparison is illustrated in

Table 2a. In this case, %OS of ZNPID seems to be very large (about 80%), which may not be acceptable in many situations. AZNPID is found to reduce the %OS by more than 65% compared to ZNPID, whereas LPID also offers a lower overshoot but with a considerably larger settling-time. Observe that, RZNPID provides 46% overshoot (Fig. 6(a)) with no improvement in the load regulation with respect to ZNPID. Even for a 50% perturbation in dead-time i.e., for L = 0.3 s, AZNPID still provides much better performance compared to

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(a) Responses and control actions of the third-order linear process with $\beta = 0.1$.

(b) Responses and control actions of the third-order linear process with $\beta=$ 0.2.

Fig. 7.

Table 3b

Stability and robustness analysis for the third-order linear process: $G_p(s) = \frac{1-0.1s}{(1+s)^3}$.

	ZNPID	AZNPID		Kharitonov's Polynomials for AZNPID with α_{max} and α_{min}
		$\alpha_{\rm max}$	α_{\min}	
GM (dB) PM (deg)	16.1 (3.39 rad/s) 39.2 (1.13 rad/s)	15.7 (4.34 rad/s) 54.6 (1.43 rad/s)	15.6 (4.4 rad/s) 54.8 (1.47 rad/s)	$s^{4} + 2.7s^{3} + 5.66s^{2} + 4.49s + 0.38 = 0;$ $s^{4} + 2.69s^{3} + 5.74s^{2} + 4.47s + 0.56 = 0;$ $s^{4} + 2.7s^{3} + 5.74s^{2} + 4.49s + 0.56 = 0;$ $s^{4} + 2.69s^{3} + 5.66s^{2} + 4.47s + 0.38 = 0$

ZNPID, RZNPID, and LPID. Performance robustness against $\pm 25\%$ variation in k_1 , k_2 , and k_3 has also been established from Table 2a. Moreover, Table 2a shows that the overall performance of AZNPI is very poor, and for a moderate change in dead-time the process becomes unstable. Table 2b clearly depicts the higher stability margins of AZNPID compared to ZNPID at the two extreme values of α while the process is under sustained oscillation. All the four Kharitonov's polynomials (Table 2b) with negative roots justify that the closed-loop system is robustly stable.

3.1.3. Third-order linear process

A second-order lag plus dead-time (SOPDT) model is usually considered to be a fair approximation for most of the industrial processes. However, to study the effectiveness of the proposed scheme, third-order linear as well as nonlinear process models are also tested. The transfer function of such a linear system is

$$G_p(s) = \frac{1 - \beta s}{(1+s)^3}.$$
(13)

Two different values of β , i.e., $\beta = 0.1$ and 0.2 are considered for this process. Fig. 7(a) and (b) show the responses and their corresponding control actions of (13) for the two separate values of β respectively. Various performance indices for different controllers are recorded in Table 3a. Like previous examples, we have tuned the controllers for $\beta = 0.1$ and studied their performances for $\beta = 0.2$ as well. In both cases, %OS of ZNPID is quite large and its load regulation is also not satisfactory. Here, LPID shows a very poor performance due to its sluggish response. In case of RZNPID, although the %OS is significantly reduced, it shows equally poor load disturbance response as ZNPID does. On the other hand, in case of AZNPID there is a remarkable improvement in the set-point response, and a considerable improvement in the load rejection behavior (Fig. 7(a) and (b)). With $\pm 25\%$ variations in tuning parameters k_1 , k_2 , and k_3 performance indices are listed in Table 3a, which indicates a robust nature of AZNPID against its parametric variations. Relative stability margins (GM and PM) for ZNPID and AZNPID are recorded in Table 3b.

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Table 4

Performance analysis of the second-order nonlinear process: $\frac{d^2y}{dt^2} + \frac{dy}{dt} + 0.2y^2 = u(t - L)$.

L		ZNPID	RZNPID	LPID	AZNPI	AZNPID							
						$k_1 = 1$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 1$	$k_1 = 1$
						$k_2 = 1$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1$	$k_2 = 1.25$	$k_2 = 1$
						$k_3 = 12$	$k_3 = 15$	$k_3 = 9$	$k_3 = 9$	$k_3 = 15$	$k_3 = 12$	$k_3 = 12$	$k_3 = 15$
	%OS	65.78	43.53	10.48	29.60	16.50	14.55	18.78	18.72	14.68	16.72	16.22	14.61
036	$t_r(s)$	1.50	1.70	2.10	2.10	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
0.5 5	$t_s(s)$	8.70	8.40	4.40	12.80	4.50	4.70	4.40	4.40	4.70	4.50	4.50	4.70
	IAE	3.66	3.32	2.60	4.34	2.68	2.69	2.67	2.67	2.70	2.68	2.68	2.70
	ITAE	27.16	25.75	21.49	46.95	22.32	22.51	22.13	22.07	22.57	22.26	22.34	22.54
	%OS	79.48	54.94	15.51	33.29	22.18	18.91	25.53	25.52	18.97	22.59	21.74	18.94
0.4 c	$t_r(s)$	1.40	1.60	2.00	2.00	1.70	1.70	1.60	1.60	1.70	1.70	1.70	1.70
0.45	$t_s(s)$	10.80	10.60	4.50	Unstable	7.80	7.70	7.80	7.80	7.70	7.80	7.80	7.70
	IAE	4.68	4.21	2.80	5.29	2.93	2.90	2.97	2.97	2.91	2.94	2.93	2.90
	ITAE	39.44	37.09	23.08	64.00	25.00	24.99	25.07	25.04	25.04	24.99	25.00	25.01









Time(sec)

Fig. 8.

3.2. Nonlinear processes

Here, we see that PM of AZNPID is higher than that of ZNPID. Kharitonov's polynomials formed with two boundary values of α (α_{max} and α_{min}) ensure the stability robustness of the close-loop system. From response curves (Fig. 7(a) and (b)), and listed performance parameters (Tables 3a and 3b) it is observed that the overall performance of AZNPID is quite good compared to other controllers.

Besides linear processes illustrated above, we also studied the performance of AZNPID for a number of nonlinear secondand third-order dead-time processes. However, since the stability analysis for nonlinear systems is not easy and our proposed AZNPID is also a nonlinear controller, it becomes very difficult to provide stability analysis for nonlinear processes. Therefore, in

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Table 5

Performance analysis of the third-order nonlinear process: $\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y^2 = u(t - L)$.

L		ZNPID	RZNPID	LPID	AZNPI	AZNPID							
						$k_1 = 1$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 0.75$	$k_1 = 1.25$	$k_1 = 1$	$k_1 = 1$
						$k_2 = 1$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1.25$	$k_2 = 0.75$	$k_2 = 1$	$k_2 = 1.25$	$k_2 = 1$
						$k_3 = 12$	$k_3 = 15$	$k_3 = 9$	$k_3 = 9$	$k_3 = 15$	$k_3 = 12$	$k_3 = 12$	$k_3 = 15$
0.2 s	%OS	55.19	18.78	0.00	0.00	0.42	0.00	4.40	4.25	0.00	1.00	0.00	0.00
	$t_r(s)$	1.10	1.50	1.70	2.50	1.40	1.50	1.30	1.30	1.50	1.40	1.40	1.50
	$t_s(s)$	5.20	4.10	10.00	5.50	3.30	2.90	3.60	3.60	2.70	3.40	3.20	2.80
	IAE	2.44	2.19	2.52	2.41	1.91	1.91	1.95	1.95	1.92	1.92	1.91	1.91
	ITAE	14.71	13.79	15.05	16.73	13.28	13.10	13.54	13.47	13.17	13.30	13.24	13.13
	%OS	73.50	32.19	5.02	0.00	12.83	8.96	16.74	16.66	9.01	13.53	12.09	8.97
036	$t_r(s)$	1.10	1.40	1.50	2.70	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20
0.5 5	$t_s(s)$	9.00	6.80	9.60	5.80	5.90	5.60	6.10	6.00	5.60	5.90	5.80	5.60
	IAE	3.62	3.06	2.66	2.73	2.31	2.26	2.39	2.38	2.26	2.33	2.30	2.26
	ITAE	25.68	23.42	16.60	23.00	17.04	16.76	17.41	17.36	16.77	17.11	16.96	16.76







Time(sec)

Fig. 9.

the present study, we could not provide stability analysis for the nonlinear processes illustrated below.

3.2.1. Second-order nonlinear process

Let us consider the nonlinear process

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 0.2y^2 = u(t-L).$$
(14)

Responses of (14) and the corresponding control actions with L = 0.3 s under ZNPID, RZNPID, and AZNPID are shown in Fig. 8(a). Though the controllers are tuned for L = 0.3 s, a higher value, i.e., L = 0.4 s is also tested without changing the controllers' parameters. Fig. 8(b) illustrates the response characteristics and the related control actions for L = 0.4 s. Table 4 presents the detailed performance indices of the various controllers. From Fig. 8(a) and

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Fig. 10. Experimental setup of feedback digital servo process rig.

(b) and Table 4 it is found that in case of AZNPID, %OS is reduced by more than 70% compared to ZNPID, and nearly 60% compared

to RZNPID with a significantly reduced value of t_s . In this case, LPID offers comparable performance with AZNPID. Performance analysis reveals that unlike ZNPID and RZNPID, AZNPID is capable of providing satisfactory result even for a reasonable change in the process dead-time. Note that for this nonlinear process AZNPI provides very poor performance (Table 4), even the process becomes unstable when the dead-time is moderately increased. Like previous case, Table 4 also shows the robust performance of AZNPID against $\pm 25\%$ changes in k_1 , k_2 , and k_3 .

3.2.2. Third-order nonlinear process

Lastly, the following third-order nonlinear process is considered

$$\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y^2 = u(t - L).$$
(15)

Fig. 9(a) and (b) respectively show the responses and control actions of (15) for two different values of dead-time L = 0.2 s and 0.3 s. Performance indices for all the controllers are depicted



(a) Response and control action under ZNPID.



(b) Response and control action under AZNPID.



(a) Response and control action under ZNPID with 25% increased K_p .



(b) Response and control action under AZNPID with 25% increased K_p.

Fig. 12.

in Table 5. Similar to the linear process in (13), for this nonlinear process too, AZNPID exhibits much better performance under both set-point change and load disturbance compared to ZNPID, RZNPID, and LPID. Although certain improvements are found in RZNPID during set-point response but no improvement is observed in load regulation. Sluggish nature of LPID results larger settling-time along with higher *ITAE* values. Table 5 indicates that except %OS, all other performance indices of AZNPID are better that those of AZNPI. As reflected in Table 5, AZNPID is found to be capable of providing almost the same level of performance with reasonable variations in k_1 , k_2 , and k_3 . Thus, once again, AZNPID exhibits improved performance over ZNPID, RZNPID, LPID, and AZNPI. At the same time, it shows performance robustness against considerable variations in the controller parameters as well as process parameter.

3.3. Real time implementation

Servo-based position control is a typical problem in various industrial processes. We have experimented on a laboratory

scale PC based DC servo motor rig (Digital Servo Workshop with MATLAB, Model: 33-004, Manufacturer: Feedback [30]) to demonstrate the performance of our proposed algorithm. The digital servo rig has two parts, namely hardware and software. The hardware units are mechanical unit (model: 33-100) and digital unit (model: 33-120). Mechanical unit consists of power amplifier, dc servo motor and tacho-generator, absolute and incremental digital encoders, input and output potentiometers, a digital speed and voltage display, and a sine, square, and triangular waveform generator. The digital unit carries ADC and DAC for signal conversion, switching, multiplexing, encoder circuits, and PWM motor drive. With the help of SIMULNIK programming under MATLAB 6.5 we implement different control algorithms.

During experimentation we have used SIMULINK generated square wave as the reference signal. The experimental setup is shown in Fig. 10. Here we have implemented our proposed controller using SIMULINK programming. With the help of Real Time Workshop (RTW) and Real Time Windows Target (RTWT) environment the performance of the proposed auto-tuning scheme

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(a) Response and control action under ZNPID with 25% reduced K_p .



(b) Response and control action under AZNPID with 25% reduced K_p .

Fig. 13.

has been implemented for achieving the desired position (given by the reference signal) by the dc servo motor. RTWT communicates with the control program, and interfaces the mechanical unit through the digital board. RTW generates C code using Microsoft C++ Professional from the SIMULNK block diagram and acts as the intermediary for two way data flow from the physical servo system to and from the SIMULNK model. The tuning of the PID controller is done according to ZN relation (proportional, integral, and derivative parameters are given by the manufacturer Feedback). Fig. 11(a) and (b) show the responses and variation of control signals for ZNPID and AZNPID, respectively. We also observe performances with $\pm 25\%$ perturbations in the proportional gain, K_p . Corresponding responses and control signals are depicted in Figs. 12 and 13. Fig. 11 reveals an improved tracking performance of AZNPID compared to ZNPID. A similar performance is observed with 25% increased K_p as depicted by Fig. 12. However, comparable performances are exhibited by ZNPID and AZNPID with 25% reduced K_p .

We may observe that the proposed scheme supposed to provide significantly improved performances for those systems which produce large overshoots under ZNPID. Simulation results (Figs. 5–9 and Tables 1a, 1b, 2a, 2b, 3a, 3b, 4 and 5) clearly substantiate this fact. However, the real servo system does not exhibit such behavior, as a result, we find marginal improvement in the performance of AZNPID over ZNPID as shown in Figs. 11–13.

4. Conclusion

A simple model independent auto-tuning scheme has been presented for ZNPIDs. The proposed scheme continuously adjusts the proportional, integral, and derivative gains through some easily interpretable heuristic rules using a single nonlinear gain adaptive parameter α , defined on the instantaneous process states. It can be easily accommodated in an existing controller. Effectiveness of the proposed AZNPID has been tested on a number of linear and nonlinear high-order dead-time processes. The performance of AZNPID has also been evaluated on a practical servo-based position control system. AZNPID has shown consistently enhanced performance both in transient and steady state conditions compared to ZNPID, RZNPID, LPID, and AZNPI. Relative stability

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for AZNPID is studied for linear systems and stability robustness has been checked through Kharitonov's polynomials. Performance robustness of AZNPID has been established by varying the process dead-time as well as its tuning parameters k_1 , k_2 , and k_3 . Moreover, the proposed scheme appeared to be a generalized one, since the same values of k_1 , k_2 , and k_3 have been used for all the processes.

In the present study, we have used empirical values of k_1 , k_2 and k_3 . Further works may be done to find their more appropriate values. Here, we have addressed the stability issues of AZNPID for linear systems only. More generalized stability analysis for linear as well as nonlinear systems may also be tried to make this study more meaningful.

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